§ Case with $\mathbb{C}$ - Jordan block :

Ex: • $A \in M_{4\times4}(\mathbb{R})$, only possible case is

$$J = \begin{pmatrix} \lambda & 1 & & \\ 0 & \lambda & & \\ & & \bar\lambda & 1 \\ & & 0 & \bar\lambda \end{pmatrix}, \quad Q = \begin{pmatrix} | & | & | & | \\ v_1, & v_2, & \bar{v}_1, & \bar{v}_2 \\ | & | & | & | \end{pmatrix}$$

with $Av_1 = \lambda v_1$, $Av_2 = \lambda v_2 + v_1$.

• $$Z(t) = e^{Jt} = \begin{pmatrix} e^{\lambda t} & te^{\lambda t} & & \\ 0 & e^{\lambda t} & & \\ & & e^{\bar\lambda t} & te^{\bar\lambda t} \\ & & 0 & e^{\bar\lambda t} \end{pmatrix}$$

• $$X(t) = Q\,Z(t) = \begin{pmatrix} | & | & | & | \\ e^{\lambda t}v_1, & e^{\lambda t}(v_2 + tv_1), & e^{\bar\lambda t}\bar{v}_1, & e^{\bar\lambda t}(\bar{v}_2 + t\bar{v}_1) \\ | & | & | & | \end{pmatrix}.$$

• **For $\mathbb{R}$-valued:**

$$X(t) = \begin{pmatrix} | & | & | & | \\ \mathrm{Re}(e^{\lambda t}v_1), & \mathrm{Re}(e^{\lambda t}(v_2+tv_1)), & \mathrm{Im}(e^{\lambda t}v_1), & \mathrm{Im}(e^{\lambda t}(v_2+tv_1)) \\ | & | & | & | \end{pmatrix}$$